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Ignition delay of non-premixed stagnation-point flows

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Abstract

Ignition delay of stagnation-point oxidizing flows over a wall with the injection of fuel is analyzed numerically. The validity of various criteria of ignition delay, i.e., the adiabaticity criterion and the thermal runaway criteria ($\partial^2 T_{\max}/\partial t^2 = 0$ and $\partial^2 \omega_{\max}/\partial t^2 = 0$), is investigated for the problems of cold flow/hot wall and hot flow/cold wall. For cold flow/hot wall systems, the ignition delay decreases with the mass flux of fuel (m_w) if m_w is below a critical value ($m_{w,c}$). The ignition delay is kinetically controlled for $m_w < m_{w,c}$. For $m_w > m_{w,c}$, the ignition delay increases with m_w and is diffusionally controlled by the deficient oxidizer. The adiabaticity criterion is suggested from the viewpoints of practice and simplicity. For hot flow/cold wall systems, the ignition delay decreases with m_w and is diffusionally controlled by the deficient fuel. The criterion of $\partial^2 \omega_{\max}/\partial t^2 = 0$ is suggested both qualitatively and quantitatively. In addition, the effects of flow strain rate, Lewis numbers and Prandtl number on ignition delay are investigated.

1. Introduction

Ignition of combustible flows has received appreciable attention because this issue is of both academic and practical interest. Most research works were concerned with the ignition criteria corresponding to the condition for lower vertical tangency on an S-shaped ignition–extinction response curve according to a steady-state model [1–13]. Based on a steady-state model, the ignition criterion only determines whether the ignition for the system of interest is eventually achieved or not; thereby the information of ignition delay is not available. Physically the solution of ignition delay is directly related to answer how long it will take to ignite the combustible mixture for a given system. From a practical point of view, the magnitude of ignition delay is frequently significant for the design of combustion system. As compared with the results of steady ignition criteria, the physical concepts of ignition delay for various combustible flows are relatively inadequate. Therefore, the relevant analyses of ignition delay are required. Ignition delay of non-premixed stagnation-point flows is investigated numerically in the present work. The problem of ignition delay is intrinsically unsteady such that a transient approach with given initial conditions is needed for the analysis of ignition delay.

For combustible stagnation-point flows, the steady ignition criteria were obtained [1–3] for various system conditions. The steady ignition criteria for other boundary-layer flows [4–9], mixing layers [10], jets [11, 12] and pipes [13] were investigated theoretically. Several research works concerned with the ignition delay including the problem of droplet ignition were published [14–20]. The ignition delay for the above wall-bounded and boundary-free shear flows was not systematically analyzed. For wall-bounded shear flows, the chemically reacting stagnation-point flow is not only a fundamentally important flow configuration but also is probably the unique one that admits self-similar solutions. Thus the stagnation-point flow is preferred on the academic side, especially for the first step in research. Ignition delay of stagnation-point oxidizing flows with a mass flux of fuel injected from the wall is of our interest. The temperature of fuel injected from the wall is the same as that of the wall. In particular, two distinct types of flow conditions are considered such as the system of cold oxidizing flows with the injection of fuel from a hot wall and that of hot oxidizing flows with the injection of fuel from a cold wall.

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Nomenclature

a_p	defined in Eq. (1)
B	frequency factor
c_p	specific heat at constant pressure
D	mass diffusivity
E	activation energy
F	defined in Eq. (10)
f	defined by $F = f'$
f'	$(= \partial f/\partial\xi)$
H	defined in Eq. (10)
Le	Lewis number
l_t	flame thickness
m_w	mass flux of fuel injected from the wall
N_2	nitrogen
n_i	reaction order
P	pressure
Pr	Prandtl number
q	specific heat of combustion
R	gas constant
R_u	universal gas constant
s_f^o	propagation speed of premixed flame with the same $\tilde{Y}_{O,\infty}$ and $\tilde{Y}_{F,P}$
T	temperature
t	time
u	velocity in the x direction
v	velocity in the y direction
W	molecular weight
x	coordinate along the wall
Y	mass fraction
y	coordinate normal to the wall

Greek symbols

α	strain rate of flow
β	temperature exponent
θ	non-dimensional activation energy
λ	thermal conductivity
μ	viscosity
ν	stoichiometric coefficient
ξ	defined in Eq. (16)
ρ	density
σ	mass stoichiometric ratio
ω	specific reaction rate

Subscripts

c	critical value
F	fuel
i	index for species
ig	ignition
max	maximum value
min	minimum value
O	oxidizer
w	wall
∞	oxidizing ambience

Superscripts

$\sim$	dimensional or original quantity
$*$	transformed coordinate

The latter case is of great interest because the ignition is achieved due to outer hot flows instead of a hot wall.

Three typical criteria of ignition are adopted in this work, i.e., the adiabaticity criterion ($\partial T/\partial y = 0$ at the wall) and the thermal runaway criteria ($\partial^2 T_{max}/\partial t^2 = 0$ and $\partial^2 \omega_{max}/\partial t^2 = 0$). The validity of these ignition criteria for both the cold flow/hot wall and the hot flow/cold wall systems is investigated. All mechanisms of ignition delay are analyzed to establish the relationships between these mechanisms of ignition delay and system parameters. The sensitivity of system parameters (e.g., the strain rate of oxidizing flows, the Lewis numbers and the Prandtl number) influencing the ignition delay is assessed. The results of ignition delay obtained here are valuable for researchers and engineers to extend their concepts of ignition.

2. Formulation

For the problem of oxidizing stagnation-point flows with a mass flux of fuel injected from the wall, the following assumptions are made. The flow is unsteady, two-dimensional and laminar viscous; the specific heats at constant pressure of the various species are equal to a constant; the radiative heat transfer, Soret and Dufour effects are neglected; the temperature of fuel injected from the wall is equal to that of the wall; chemical reactions between the fuel (F) and the oxidizer (O) are represented by a global one-step irreversible reaction ($F + \nu_O O \rightarrow$ products); the combustible gas is a mixture of ideal gases with constant values of $\tilde{\rho}\tilde{\mu}$, $\tilde{\rho}\tilde{\lambda}$ and $\tilde{\rho}^2\tilde{D}_i$. The nitrogen gas ($N_2$) is frequently an abundant species in the gaseous mixture of oxidizing flow (e.g., air). We consider that the fuel diluted by N_2 is injected from the porous wall. For simplicity, the universal gas constant $\tilde{R}_u$ in the ideal-gas equation of state is replaced by the gas constant $\tilde{R}$ where $\tilde{R} = \tilde{R}_u/\tilde{W}_{N_2}$ and the transport coefficients of the gaseous mixture $\tilde{\mu}$, $\tilde{\lambda}$ and $\tilde{D}_i$ are approximated by $\tilde{\mu}_{N_2}$, $\tilde{\lambda}_{N_2}$ and $\tilde{D}_{i,N_2}$, respectively.

The origin of the coordinate system is located at the wall in a stagnation-point flow, with x and y indicating the distances along and normal to the wall, respectively.

With the above assumptions, in terms of the following non-dimensional variables:

$$\rho=\tilde{\rho}/\tilde{\rho}_\infty,\quad u=\tilde{u}/\tilde{s}_\mathrm{f}^\mathrm{o},\quad v=\tilde{v}/\tilde{s}_\mathrm{f}^\mathrm{o},\quad x=\tilde{x}/\tilde{l}_\mathrm{t},$$
$$y=\tilde{y}/\tilde{l}_\mathrm{t},\quad t=\tilde{s}_\mathrm{f}^\mathrm{o}\tilde{t}/\tilde{l}_\mathrm{t},\quad P=\tilde{P}/\tilde{P}_\infty,\quad T=\tilde{T}/\tilde{T}_\infty,$$
$$Y_\mathrm{F}=\tilde{Y}_\mathrm{F},\quad Y_\mathrm{O}=\tilde{Y}_\mathrm{O}/\sigma,\quad \mu=\tilde{\mu}/\tilde{\mu}_\infty,\quad \lambda=\tilde{\lambda}/\tilde{\lambda}_\infty,$$
$$D_\mathrm{i}=\tilde{D}_\mathrm{i}/\tilde{D}_{\mathrm{i},\infty},\quad q=\tilde{q}/\tilde{c}_\mathrm{P}\tilde{T}_\infty,\quad \theta=\tilde{E}/\tilde{R}_\mathrm{u}\tilde{T},$$
$$a_\mathrm{P}=\tilde{P}_\infty/\tilde{\rho}_\infty\tilde{s}_\mathrm{f}^{\mathrm{o}^2},\quad Pr=\tilde{c}_\mathrm{P}\tilde{\mu}_\infty/\tilde{\lambda}_\infty,\quad Le_\mathrm{i}=\tilde{\lambda}_\infty/\tilde{\rho}_\infty\tilde{c}_\mathrm{P}\tilde{D}_{\mathrm{i},\infty},\tag{1}$$

where $\tilde{l}_\mathrm{t}=\tilde{\lambda}_\infty/\tilde{\rho}_\infty\tilde{c}_\mathrm{P}\tilde{s}_\mathrm{f}^\mathrm{o}$ and $\sigma=\nu_\mathrm{O}\tilde{W}_\mathrm{O}/\tilde{W}_\mathrm{F}$ the governing equations for the low-Mach-number stagnation-point flows are:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0,\tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-a_\mathrm{P}\frac{\partial P}{\partial x}+Pr\left\{\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial x}\left[\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\},\tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-a_\mathrm{P}\frac{\partial P}{\partial y}+Pr\left\{\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}\right)-\frac{\partial}{\partial y}\left[\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\},\tag{4}$$

$$\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+q\omega,\tag{5}$$

$$\rho\left(\frac{\partial Y_\mathrm{i}}{\partial t}+u\frac{\partial Y_\mathrm{i}}{\partial x}+v\frac{\partial Y_\mathrm{i}}{\partial y}\right)=Le_\mathrm{i}^{-1}\left[\frac{\partial}{\partial x}\left(\rho D_\mathrm{i}\frac{\partial Y_\mathrm{i}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\rho D_\mathrm{i}\frac{\partial Y_\mathrm{i}}{\partial y}\right)\right]-\omega,\quad i=\mathrm{F},\mathrm{O},\tag{6}$$

$$\rho T=1,\tag{7}$$

where $\omega=BT^{\beta-n}Y_\mathrm{F}^{n_\mathrm{F}}Y_\mathrm{O}^{n_\mathrm{O}}\exp(-\theta/T)$ with

$$B=\frac{\tilde{B}\nu_\mathrm{O}^{n_\mathrm{O}}\tilde{\lambda}_\infty}{\tilde{c}_\mathrm{P}\tilde{\rho}_\infty^2\tilde{s}_\mathrm{f}^{\mathrm{o}^2}\tilde{W}_\mathrm{F}^{n-1}\tilde{T}_\infty^{n-\beta}}\left(\frac{\tilde{P}_\infty}{\tilde{R}}\right)^n$$

and $n=n_\mathrm{F}+n_\mathrm{O}$. The relevant boundary conditions are:

at $y=0$:

$$u=0,\quad \rho v=m_\mathrm{w},\quad T=T_\mathrm{w},$$
$$\rho D_\mathrm{F}\partial Y_\mathrm{F}/\partial y=Le_\mathrm{F}m_\mathrm{w}(Y_\mathrm{F}-Y_{\mathrm{F,P}}),$$
$$\rho D_\mathrm{O}\partial Y_\mathrm{O}/\partial y=Le_\mathrm{O}m_\mathrm{w}Y_\mathrm{O},\tag{8}$$

as $y\to\infty$:

$$u\to\alpha x,\quad v\to-\alpha y,\quad \rho\to1,\quad T\to1,\quad Y_\mathrm{F}\to0,$$
$$Y_\mathrm{O}\to Y_{\mathrm{O},\infty},\tag{9}$$

where $m_\mathrm{w}=\tilde{m}_\mathrm{w}/\tilde{\rho}_\infty\tilde{s}_\mathrm{f}^\mathrm{o}$ and $\alpha=\tilde{\lambda}_\infty\tilde{\alpha}/\tilde{\rho}_\infty\tilde{c}_\mathrm{P}\tilde{s}_\mathrm{f}^{\mathrm{o}^2}$. The initial conditions are given by the steady solutions of isothermal stagnation-point flows with $T=1$, $Y_\mathrm{F}=0$ and $Y_\mathrm{O}=Y_{\mathrm{O},\infty}$.

For stagnation-point flows, we have the similar solutions as follows:

$$\rho=\rho(y,t),\quad u=xF(y,t),\quad v=v(y,t),$$
$$T=T(y,t),\quad P=-\alpha^2x^2/2a_\mathrm{P}+H(y,t),$$
$$\mu=\mu(T)=\mu(y,t),\quad \lambda=\lambda(T)=\lambda(y,t),$$
$$D_\mathrm{i}=D_\mathrm{i}(T)=D_\mathrm{i}(y,t).\tag{10}$$

Substituting the above similar solutions, we obtain

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial y}+\rho F=0,\tag{11}$$

$$\rho\left(\frac{\partial F}{\partial t}+v\frac{\partial F}{\partial y}\right)=-\rho F^2+\alpha^2+Pr\frac{\partial}{\partial y}\left(\mu\frac{\partial F}{\partial y}\right),\tag{12}$$

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial H}{\partial y}+Pr\left\{\mu\frac{\partial F}{\partial y}+2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)-\frac{2}{3}\frac{\partial}{\partial y}\left[\mu\left(F+\frac{\partial v}{\partial y}\right)\right]\right\},\tag{13}$$

$$\rho\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+q\omega,\tag{14}$$

$$\rho\left(\frac{\partial Y_\mathrm{i}}{\partial t}+v\frac{\partial Y_\mathrm{i}}{\partial y}\right)=Le_\mathrm{i}^{-1}\frac{\partial}{\partial y}\left(\rho D_\mathrm{i}\frac{\partial Y_\mathrm{i}}{\partial y}\right)-\omega,\quad i=\mathrm{F},\mathrm{O}.\tag{15}$$

For the convenience of numerical computation, we introduce the transformed coordinate system (x^*,ξ,t^*), where

$$x^*=x,\quad \xi(y,t)=\int_0^y\rho(y',t)\,\mathrm{d}y'\text{ and }t^*=t.\tag{16}$$

The continuity equation (11) is automatically satisfied for $F=f'\equiv\partial f/\partial\xi$ and $v=-(f+\partial\xi/\partial t)/\rho$. Eqs. (12), (14) and (15) become

$$\partial f'/\partial t^*-ff''=-f'^2+\alpha^2T+Prf''',\tag{17}$$

$$\frac{\partial T}{\partial t^*}-f\frac{\partial T}{\partial\xi}=\frac{\partial^2T}{\partial\xi^2}+qT\omega,\tag{18}$$

$$\frac{\partial Y_\mathrm{i}}{\partial t^*}-f\frac{\partial Y_\mathrm{i}}{\partial\xi}=Le_\mathrm{i}^{-1}\frac{\partial^2Y_\mathrm{i}}{\partial\xi^2}-T\omega,\quad i=\mathrm{F},\mathrm{O},\tag{19}$$

with $\rho\mu=\rho\lambda=\rho^2D_\mathrm{i}=1$ due to the constant values of $\tilde{\rho}\tilde{\mu}$, $\tilde{\rho}\tilde{\lambda}$ and $\tilde{\rho}^2\tilde{D}_\mathrm{i}$. The boundary conditions (8) and (9) are reduced to be:

at $\xi=0$:

$$f'=0,\quad f=-m_\mathrm{w},\quad T=T_\mathrm{w},$$
$$\partial Y_\mathrm{F}/\partial\xi=Le_\mathrm{F}m_\mathrm{w}(Y_\mathrm{F}-Y_{\mathrm{F,P}}),\quad \partial Y_\mathrm{O}/\partial\xi=Le_\mathrm{O}m_\mathrm{w}Y_\mathrm{O},\tag{20}$$

as $\xi \to \infty$:

$$f' = \alpha, \quad T \to 1, \quad Y_F \to 0, \quad Y_O \to Y_{O,\infty}. \tag{21}$$

Finally, the problem of interest is governed by Eqs. (17)–(19) with the boundary conditions (20) and (21). A numerical analysis is performed to obtain the ignition delay for varied system parameters.

3. Criterion of ignition delay

There are two types of criteria of ignition delay for the problem of interest, i.e., the adiabaticity criterion and the thermal runaway criterion. The definition of adiabaticity criterion is $\partial T/\partial y = 0$ at the wall, where the direction y is normal to the wall. The adiabaticity criterion is readily applied to the problems of thermal ignition with a hot wall. It implies that the heat release of chemical reactions is so much that the heat starts to be transferred to the hot boundary instead of receiving the heat from it. Physically, the chemical reaction leading to ignition has become self-sustaining. This criterion can be approximately considered to be $\partial\omega/\partial y = 0$ at the wall because the dependence of chemical reaction on temperature is great and the consumption of reactants prior to ignition defined by $\partial T/\partial y = 0$ at the wall is small. According to the criterion of $\partial\omega/\partial y = 0$ at the wall, a transition of the location of maximum chemical reaction rate from the hot solid boundary to the gas phase occurs as this criterion is achieved. The magnitudes of ignition delay predicted by both $\partial T/\partial y = 0$ and $\partial\omega/\partial y = 0$ at the wall are quite close.

Traditionally, there are two ways to define the thermal runaway criterion, i.e., $\partial^2 T_{max}/\partial t^2 = 0$ and $\partial^2\omega_{max}/\partial t^2 = 0$. The criterion of $\partial^2\omega_{max}/\partial t^2 = 0$ is directly related to the actual burning state of flames; thereby it is better than the criterion of $\partial^2 T_{max}/\partial t^2 = 0$ both physically and theoretically. The numerical results presented later also reveal that the criterion of $\partial^2\omega_{max}/\partial t^2 = 0$ instead of that of $\partial^2 T_{max}/\partial t^2 = 0$ is suggested both qualitatively and quantitatively even though the criterion of $\partial^2\omega_{max}/\partial t^2 = 0$ is difficult to be determined by experiment due to the lack of instantaneous measurement of ω_{max}. The thermal runaway criterion indicates that a steady flame will be established when the maximum reaction rate rises rapidly and then gradually levels off. As compared with the adiabaticity criterion, it is more favorable to determine the ignition delay of non-premixed systems composed of hot oxidizing flows over a cold wall with the injection of fuel according to the thermal runaway criterion.

4. Numerical method

The method of lines [21] is adopted to solve Eqs. (17)–(19) with the boundary conditions (20) and (21). All spatial derivatives are discretized according to a power law [22] whereas the time derivative remains continuous. The magnitude of f is considered as a given solution obtained readily by integrating f' from 0 to ξ at the previous time step. Thus the problem of interest is reduced to a system of coupled non-linear ordinary differential equations. The fourth-order accurate Runge–Kutta–Fehlberg scheme with a local integration error equal to 10^{-5} is used in this work. The infinite domain $0 \leqslant \xi < \infty$ is truncated in such a way that the numerical solutions are not influenced by the boundary of computational domain which is $\xi = 20.0$ in this work. The grid size is adjusted to obtain an accurate solution of ignition delay with a maximum relative error equal to 3%. Depending on system parameters, the range of the number of grid points is from 301 to 451.

5. Results and discussion

The input system parameters in the present work are $\tilde{B} = 7.4 \times 10^{11}$, $\beta = 0.0$, $n_F = 0.15$, $n_O = 1.6$, $\tilde{P} = 1$ atm, $\tilde{c}_p = 1.05 \times 10^{-3}$ kJ/g K, $\tilde{q} = 42$ kJ/g, $\tilde{W}_F = 58$ g/mol, $\tilde{W}_O = 32$ g/mol, $\nu_O = 6.5$, $Y_{O,\infty} = 0.06$ and $\theta = 50.0$.

5.1. Effects of wall temperature, mass flux of fuel and temperature of oxidizing flow on ignition delay

According to the adiabaticity criterion ($\partial T/\partial y = 0$ at $y = 0$), the plot of ignition delay (t_{ig}) versus the mass flux of fuel injected from the hot wall (m_w) for varied wall temperature (T_w) is shown in Fig. 1. The results reveal that the ignition delay decreases initially with m_w and

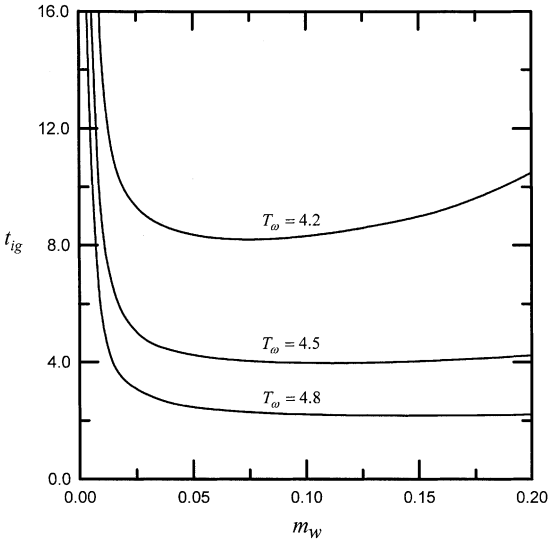

Fig. 1. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied T_w according to the adiabaticity criterion ($T_\infty = 1.0$, $\alpha = 0.01$, $Y_{F,P} = 0.4$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

then increases with m_w after the magnitude of m_w reaches a critical value ($m_{w,c}$). The minimum ignition delay occurs at $m_w = m_{w,c}$. For $m_w < m_{w,c}$, the ignition delay decreases with m_w such that the kinetically controlled ignition delay is physically expected. The gas-phase temperature near the wall increases with the mass flux of hot fuel injected from it. The chemical reactions leading to ignition are mainly influenced by temperature for the region of kinetically controlled ignition ($m_w < m_{w,c}$). Physically, the temperature in the vicinity of wall increases gradually with the mass flux of fuel injected from the hot wall, but the concentration of oxidizer near the wall decreases with it. The ignition delay eventually becomes diffusionally controlled due to the deficient oxidizer when the mass flux of fuel injected from the wall exceeds a certain critical value ($m_{w,c}$). A transition between the kinetically controlled ignition delay and the diffusionally controlled ignition delay occurs at $m_w = m_{w,c}$. The effective Lewis number is expected to be Le_O, especially for the region of diffusionally controlled ignition. As shown in Fig. 1, the region of kinetically controlled ignition increases with the wall temperature. The kinetically controlled ignition delay is preferred for great T_w because of an exponential dependence of chemical reactions on temperature according to the Arrhenius rate expression. The magnitudes of $m_{w,c}$ for $T_w = 4.2$, 4.5 and 4.8 are 0.065, 0.11 and 0.14, respectively. For the great wall temperature (e.g., $T_w = 4.8$), the characteristics of diffusionally controlled ignition delay are not obvious such that the corresponding magnitudes of ignition delay can be roughly viewed as a constant, which depends on T_w.

According to two types of thermal runaway criteria ($\partial^2 T_{max}/\partial t^2 = 0$ and $\partial^2 \omega_{max}/\partial t^2 = 0$), the results of t_{ig} versus m_w injected from the hot wall for varied T_w are illustrated in Fig. 2. As compared with the magnitudes of t_{ig} given in Fig. 1, the ignition delay is modified mainly in a quantitative way. The ignition delay shown in Fig. 2 is invariably greater than that shown in Fig. 1. The characteristics of diffusionally controlled ignition delay become obvious for great wall temperature. The qualitative behaviors of ignition delay are almost the same for these two criteria of ignition delay. The ignition delay obtained by $\partial^2 \omega_{max}/\partial t^2 = 0$ is smaller than that obtained by $\partial^2 T_{max}/\partial t^2 = 0$. According to Fig. 2, the quantitative difference of $m_{w,c}$ between two ignition criteria is negligible. The magnitudes of $m_{w,c}$ for $T_w = 4.2$, 4.5 and 4.8 are 0.07, 0.12 and 0.155, respectively. As compared with the results in Fig. 1, the region of diffusionally controlled ignition caused by the deficient oxidizer slightly decreases because the concentration of oxidizer at the location of ignition, where it keeps away from the wall according to the thermal runaway criterion, is greater than that at the wall.

Physically, there is a minimum value of m_w ($m_{w,min}$) below which the ignition of non-premixed flames for the problem of interest cannot be achieved. The ignition delay approaches an infinite value at $m_w = m_{w,min}$, as shown in Figs. 1 and 2. This minimum value is a function of T_w for a given flow system [9]. Obviously, the minimum mass flux of fuel injected from the hot wall is expected to decrease as the wall temperature increases. The numerical results also reveal that the magnitude of $m_{w,min}$ is quite insensitive to the ignition criteria, namely the adiabaticity criterion, $\partial^2 \omega_{max}/\partial t^2 = 0$ and $\partial^2 T_{max}/\partial t^2 = 0$, adopted in this work. According to Figs. 1 and 2, the values of $m_{w,min}$ are 0.007, 0.0046 and 0.0035 for $T_w = 4.2$, 4.5 and 4.8, respectively.

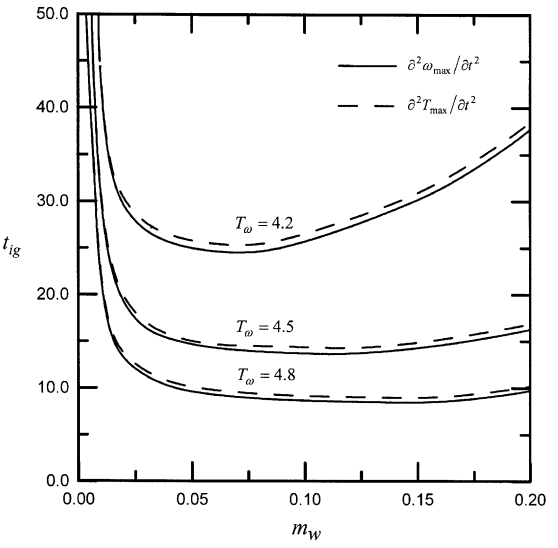

Fig. 2. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied T_w according to $\partial^2 \omega_{max}/\partial t^2 = 0$ and $\partial^2 T_{max}/\partial t^2 = 0$ ($T_\infty = 1.0$, $\alpha = 0.01$, $Y_{F,P} = 0.4$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

For the problem of thermal ignition with a hot wall, there are no explicit reasons to conclude which one of ignition criteria is better even though some quantitative discrepancies of ignition delay between the adiabatic criterion and the thermal runaway criteria are found. For the purpose of qualitative prediction, the adiabaticity criterion is suggested because the corresponding ignition delay is easier to be obtained both numerically and experimentally.

For the problem of ignition delay of hot oxidizing flows over the cold wall with an injected mass flux of fuel, the thermal energy leading to gas-phase ignition is supplied from the hot oxidizing flow itself instead of the wall. Therefore, the adiabaticity criterion is not adopted here. The ignition delay is determined appropriately by the thermal runaway criteria. The ignition delay as a function of the mass flux of fuel injected from the cold wall for varied temperature of oxidizing flow (T_∞) is shown in Fig. 3. The results reveal that the ignition delay predicted by $\partial^2 T_{max}/\partial t^2 = 0$ is greater than that

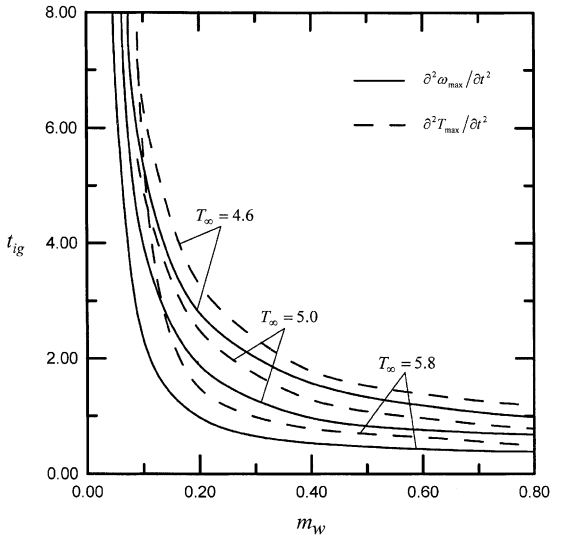

Fig. 3. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied T_∞ according to $\partial^2\omega_{max}/\partial t^2 = 0$ and $\partial^2 T_{max}/\partial t^2 = 0$ ($T_w = 1.0$, $\alpha = 0.02$, $Y_{F,P} = 0.2$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

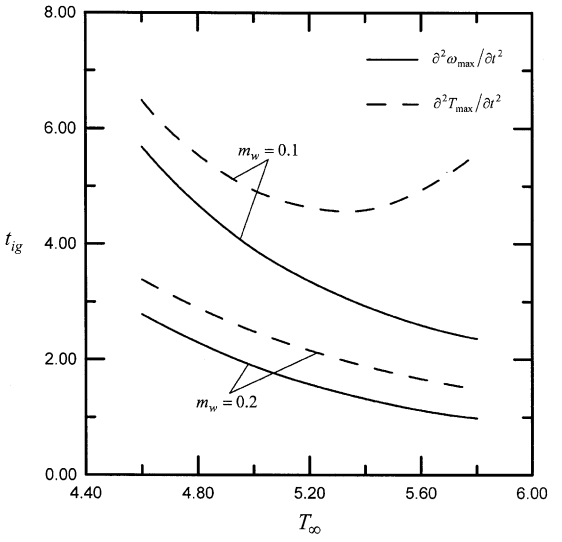

Fig. 4. Ignition delay t_{ig} versus temperature of oxidizing flow T_∞ for varied m_w according to $\partial^2\omega_{max}/\partial t^2 = 0$ and $\partial^2 T_{max}/\partial t^2 = 0$ ($T_w = 1.0$, $\alpha = 0.02$, $Y_{F,P} = 0.2$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

predicted by $\partial^2\omega_{max}/\partial t^2 = 0$. For this type of ignition, the thermal ignition is expected to occur at a location near the side of hot oxidizer due to the great dependence of chemical reactions on temperature. The fuel–lean combustible mixture is found at the location of ignition, and thereby the effective Lewis number is Le_F here. According to Fig. 3, the ignition delay decreases monotonically with m_w for fixed T_∞. Obviously, this fact implies that the ignition delay is diffusionally controlled by the deficient fuel at the location of ignition far away from the wall because the concentration of fuel increases with m_w. The kinetically controlled ignition delay is not observed. The cooling effect due to an increase in the mass flux of cold fuel is insignificant because a great amount of hot oxidizer is mixed with a small amount of cold fuel at the location of ignition.

According to Fig. 3, the ignition delay determined by $\partial^2\omega_{max}/\partial t^2 = 0$ decreases with T_∞ for given m_w, as physically expected. However, the ignition delay obtained by $\partial^2 T_{max}/\partial t^2 = 0$ exhibits a different result for small values of m_w (e.g., $m_w = 0.1$). The magnitudes of $m_{w,min}$ predicted by $\partial^2 T_{max}/\partial t^2 = 0$ are greater than those predicted by $\partial^2\omega_{max}/\partial t^2 = 0$. The variations of ignition delay with T_∞ for selected values of m_w according to the thermal runaway criteria ($\partial^2 T_{max}/\partial t^2 = 0$ and $\partial^2\omega_{max}/\partial t^2 = 0$) are illustrated in Fig. 4. For great m_w ($m_w = 0.2$), the ignition delay decreases monotonically with T_∞ for both thermal runaway criteria. However, the ignition delay for $m_w = 0.1$ according to the criterion of $\partial^2 T_{max}/\partial t^2 = 0$ initially decreases with T_∞, and then increases with it after the magnitude of T_∞ is greater than a critical value. Physically, the ignition delay for fixed m_w is expected to decrease with T_∞. According to the criterion of $\partial^2 T_{max}/\partial t^2 = 0$, this criterion can be satisfied only when T_{max} is greater than T_∞ because the high temperature T_∞ is the maximum system temperature initially. This fact implies that the ignition delay tends to increase for great T_∞ because the system must wait until the condition of $T_{max} > T_\infty$ is achieved. The location of flame moves toward the side of cold fuel when the mass flux of fuel injected from the wall gradually decreases. Obviously, the flame temperature decreases as m_w decreases. This mathematical restriction ($T_{max} > T_\infty$) is not suitable when the flame temperature eventually achieved is close to the temperature of hot oxidizing flow. The behavior of t_{ig} versus T_∞ for $m_w = 0.1$ according to the criterion of $\partial^2 T_{max}/\partial t^2 = 0$ (Fig. 4) is caused mathematically, not physically. With decreasing m_w further, the criterion of $\partial^2 T_{max}/\partial t^2 = 0$ becomes invalid when the flame temperature is equal to or even smaller than T_∞ for $m_w \leqslant m_{w,T_{max}=T_\infty}$. The magnitudes of $m_{w,T_{max}=T_\infty}$ here cannot be physically viewed as a minimum mass flux of fuel at ignition even though the infinite values of ignition delay are obtained in Fig. 3. Because the criterion of $\partial^2 T_{max}/\partial t^2 = 0$ fails to predict the qualitative behavior of ignition delay for small m_w (Fig. 4) and the minimum mass flux of fuel at ignition (Fig. 3), it serves only as a sufficient but not a necessary condition.

There is no such a mathematical restriction imposed on the criterion of $\partial^2\omega_{max}/\partial t^2 = 0$ when the maximum chemical reaction rate continuously increases with time. Physically, the magnitude of ω_{max} is directly related to the burning rate for the problem of interest. The numerical solutions in Figs. 3 and 4 according to the

criterion of $\partial^2\omega_{max}/\partial t^2 = 0$ are physical results. As shown in Fig. 3, the magnitudes of $m_{w,min}$ for $T_\infty = 4.6$, 5.0 and 5.8 are 0.07, 0.055 and 0.04, respectively.

5.2. Effect of flow strain rate on ignition delay

For the system of cold oxidizing flows over a hot wall with the injection of fuel, the plot of ignition delay versus flow strain rate (α) is presented in Fig. 5. The ignition delay increases with α because of an increase in the strain rate of cold flows. The effect of α on ignition delay for the region of kinetically controlled ignition ($m_w = 0.05$) is slightly greater than that for the region of diffusionally controlled ignition ($m_w = 0.2$).

For the system of hot oxidizing flows over a cold wall with the injection of fuel, the effect of α on ignition delay is shown in Fig. 6. Obviously, the ignition delay decreases with the strain rate of hot oxidizing flows. According to Fig. 6, the influence of α on ignition delay decreases as the mass flux of cold fuel increases.

5.3. Effects of Lewis number and Prandtl number on ignition delay

For the problem of cold oxidizing flows over a hot wall with the injection of fuel, the effects of Le_F and Le_O on ignition delay are shown in Figs. 7 and 8. The combustible gas at the location of ignition is a fuel-rich mixture because the ignition process proceeds just on the hot wall. Therefore, the influence of Le_O on ignition delay is greater than that of Le_F on it, as shown in Figs. 7 and 8. According to Fig. 7, the ignition delay decreases with Le_F. Physically, the mass diffusivity of fuel decreases with Le_F such that the concentration of fuel injected from the wall reaches a greater value near the wall for a smaller mass diffusivity of fuel. It is expected that the ignition delay decreases with the concentration of fuel near the wall. As shown in Fig. 7, the effect of Le_F on $m_{w,c}$ is negligible because the diffusionally controlled ignition delay is caused by the deficient oxidizer.

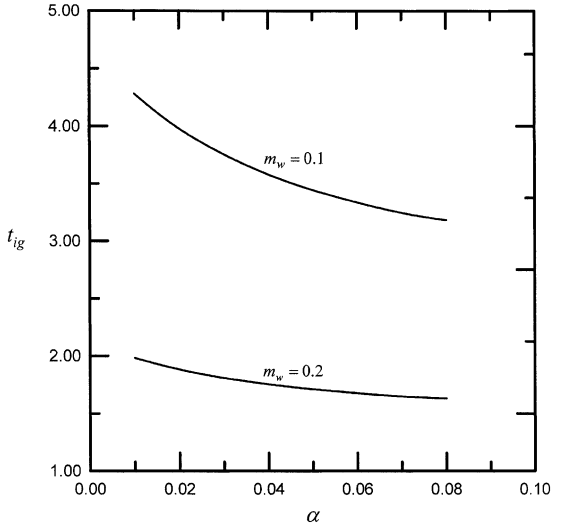

Fig. 6. Influence of flow strain rate α on ignition delay t_{ig} for hot oxidizing flows over a cold wall according to $\partial^2\omega_{max}/\partial t^2 = 0$ ($T_w = 1.0$, $T_\infty = 5.0$, $Y_{F,P} = 0.2$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

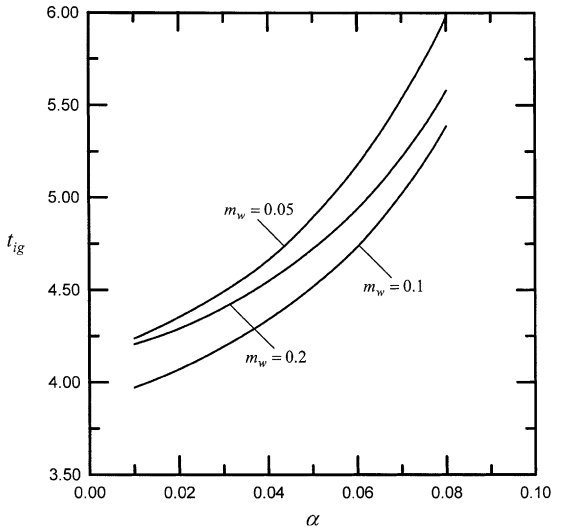

Fig. 5. Influence of flow strain rate α on ignition delay t_{ig} for cold oxidizing flows over a hot wall according to the adiabaticity criterion ($T_w = 4.5$, $T_\infty = 1.0$, $Y_{F,P} = 0.4$, $Le_F = Le_O = 1.0$ and $Pr = 0.7$).

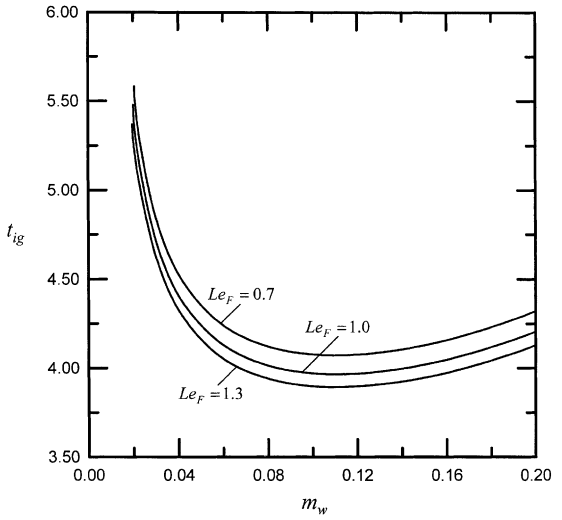

Fig. 7. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied Le_F according to the adiabaticity criterion ($T_w = 4.5$, $T_\infty = 1.0$, $\alpha = 0.01$, $Y_{F,P} = 0.4$, $Le_O = 1.0$ and $Pr = 0.7$).

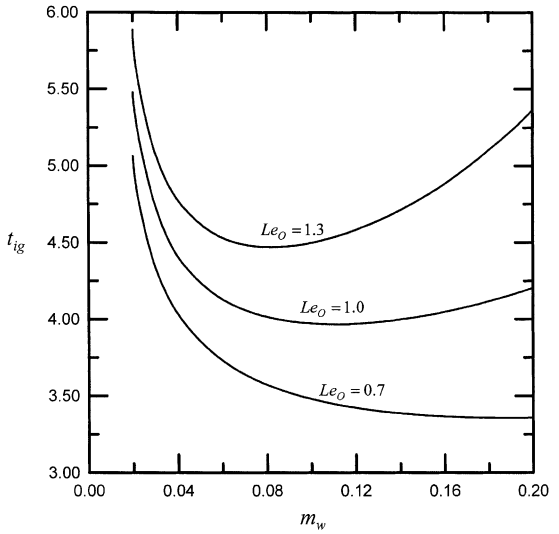

Fig. 8. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied Le_O according to the adiabaticity criterion ($T_w = 4.5$, $T_\infty = 1.0$, $\alpha = 0.01$, $Y_{F,P} = 0.4$, $Le_F = 1.0$ and $Pr = 0.7$).

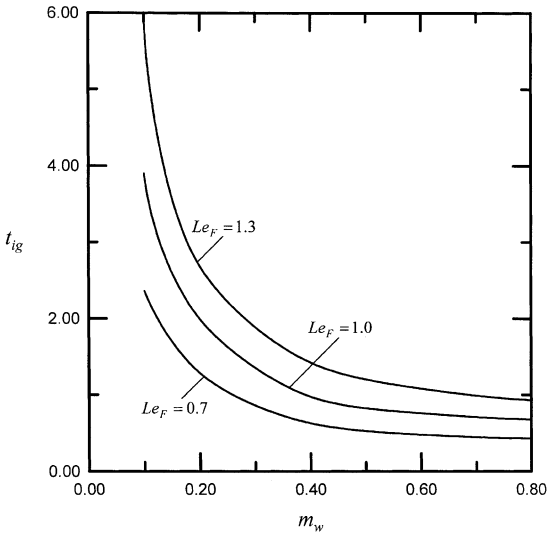

Fig. 9. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied Le_F according to $\partial^2\omega_{max}/\partial t^2 = 0$ ($T_w = 1.0$, $T_\infty = 5.0$, $\alpha = 0.02$, $Y_{F,P} = 0.2$, $Le_O = 1.0$ and $Pr = 0.7$).

According to Fig. 8, the ignition delay increases with Le_O. The concentration of oxidizer becomes smaller near the wall when the mass diffusion of oxidizer from the free stream to the wall is more difficult due to a smaller mass diffusivity of oxidizer. The effect of Le_O on $m_{w,c}$ is prominent because of diffusionally controlled ignition delay caused by the deficient oxidizer. The region of diffusionally controlled ignition increases with Le_O, as physically expected.

For the problem of hot oxidizing flows with the cold fuel injected from the wall, the effects of Le_F and Le_O on ignition delay are illustrated in Figs. 9 and 10. As discussed previously, the ignition process of fuel-lean mixtures is observed. The ignition delay is diffusionally controlled by the deficient species of fuel; thereby the effective Lewis number is Le_F. According to Figs. 9 and 10, the influence of Le_F on ignition delay is much greater than that of Le_O on ignition delay, as physically expected. For given m_w, the ignition delay increases substantially with Le_F, but decreases slightly with Le_O. The concentration of fuel near the location of ignition increases with the mass diffusivity of fuel such that a short ignition delay is preferred for small Le_F (Fig. 9). However, the concentration of oxidizer near the location of ignition decreases with the mass diffusivity of oxidizer. The long ignition delay is expected for small Le_O (Fig. 10).

As compared with the effects of mass diffusivities of reactants (Lewis numbers) on ignition delay, the influence of thermal conductivity (Prandtl number) on ignition delay is physically understandable. For the problem of cold oxidizing flows over a hot wall with the injection of fuel, the numerical results reveal that the ignition

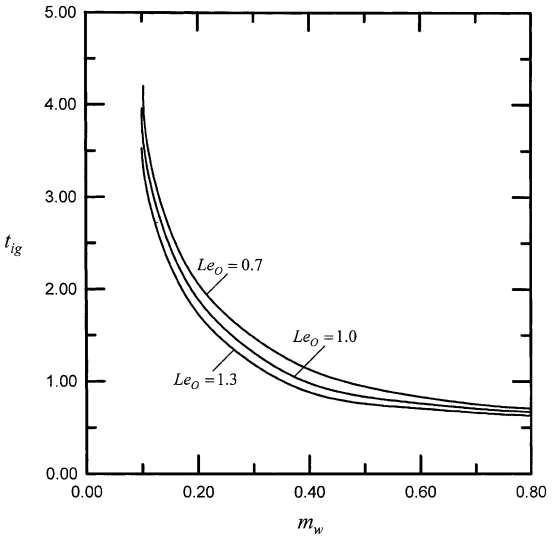

Fig. 10. Ignition delay t_{ig} versus injected mass flux of fuel m_w for varied Le_O according to $\partial^2\omega_{max}/\partial t^2 = 0$ ($T_w = 1.0$, $T_\infty = 5.0$, $\alpha = 0.02$, $Y_{F,P} = 0.2$, $Le_F = 1.0$ and $Pr = 0.7$).

delay increases with the thermal diffusivity of gaseous mixture. The region of kinetically controlled ignition decreases with the thermal conductivity because more heat loss from the hot wall to the cold flow occurs for a greater thermal conductivity. Similar to the above results, the ignition delay for hot oxidizing flows over a cold wall with the injection of fuel increases as the thermal diffusivity of gaseous mixture increases.

6. Conclusions

Ignition delay of stagnation-point oxidizing flows with a mass flux of fuel injected from the wall is investigated numerically in the present work. Three criteria of ignition delay, i.e., the adiabaticity criterion, $\partial^2 T_{max}/\partial t^2 = 0$ and $\partial^2 \omega_{max}/\partial t^2 = 0$, are adopted to evaluate the magnitudes of ignition delay for various system parameters. The following conclusions have been drawn.

For the system of cold oxidizing flows over a hot wall with the injection of fuel, the qualitative behaviors of ignition delay can be satisfactorily predicted by any of ignition criteria used in this work. The ignition delay is kinetically controlled for $m_w < m_{w,c}$. The ignition delay decreases with m_w in the region of kinetically controlled ignition. The ignition delay becomes diffusionally controlled due to the deficient oxidizer at the location of ignition for $m_w > m_{w,c}$. The ignition delay increases with m_w in the region of diffusionally controlled ignition. The ignition delay is minimum at $m_w = m_{w,c}$. The ignition delay obtained by the adiabaticity criterion is smaller than that obtained by the thermal runaway criteria. The ignition delay determined by the criterion of $\partial^2 \omega_{max}/\partial t^2 = 0$ is slightly smaller than that determined by the criterion of $\partial^2 T_{max}/\partial t^2 = 0$. Both the magnitudes of $m_{w,c}$ and $m_{w,min}$ predicted by three criteria of ignition delay here are very close. For a practical point of view, the quantitative variations of $m_{w,c}$ and $m_{w,min}$with the criteria of ignition delay are negligible. The adiabaticity criterion is suggested for the simplicity of numerical computation and experimental measurement. The ignition delay increases with the strain rate of oxidizing flows. The influence of strain rate on the ignition delay for the kinetically controlled region is slightly greater than that for the diffusionally controlled region. The effect of Le_O on ignition delay is greater than that of Le_F on it. The magnitudes of ignition delay increase and decrease with Le_O and Le_F, respectively. The ignition delay increases with the thermal conductivity. However, the region of kinetically controlled ignition decreases with it.

For the system of hot oxidizing flows over a cold wall with the injection of fuel, the ignition delay decreases with m_w for the entire range of m_w. The ignition delay is diffusionally controlled by the deficient fuel. The ignition delay is readily determined by the thermal runaway criteria instead of the adiabaticity criterion. The ignition delay obtained by the criterion of $\partial^2 \omega_{max}/\partial t^2 = 0$ is smaller than that obtained by the criterion of $\partial^2 T_{max}/\partial t^2 = 0$. According to the criterion of $\partial^2 T_{max}/\partial t^2 = 0$, the qualitative behavior of ignition delay for small m_w is not physically correct. The criterion of $\partial^2 T_{max}/\partial t^2 = 0$ is not proper to determine the minimum mass flux of fuel at ignition. This fact indicates that the magnitudes of $m_{w,min}$ are obtained only by the criterion of $\partial^2 \omega_{max}/\partial t^2 = 0$. The criterion of $\partial^2 \omega_{max}/\partial t^2 = 0$ is suggested both qualitatively and quantitatively. The ignition delay decreases with the strain rate of oxidizing flows. The effect of flow strain rate on ignition delay decreases with m_w. The influence of Le_F on ignition delay is much greater than that of Le_O on it. The values of ignition delay increase and decrease with Le_F and Le_O, respectively. The ignition delay increases with the thermal conductivity.

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